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FRACTALS AS JULIA AND MANDELBROT SETS OF LOGARITHMIC FUNCTION USING DOGAN AND KARAKAYA (DK) ITERATIVE SCHEME

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ABSTRACT

Fractals represent the phenomena of expanding symmetries which exhibit similar patterns for different scales. In this paper, we establish an escape criteria by using Dogan and Karakaya (DK) iterative process to generate fractals namely Julia and Mandelbrot sets for the logarithmic function $F(z) = log(1 + z^p) + c$, where $c \in \mathbb{C}$ and $p \geq 2$. Our result is a generalization of the existing algorithm and technique providing fractals for different parameter values. Also, the time taken to obtain fractals for different parameters by using computer software MATLAB is computed in seconds.

Keywords: Fractals, Logarithmic function, Julia set, Mandelbrot set, escape criteria

1. INTRODUCTION

Fractal patterns are very common in nature like crystals, rivers, tree branches, coast lines, electricity, clouds and so on[1]. Fractals play an important role in understanding natural or living phenomena like microorganisms [2]. Fractals are also used in mechanics to understand the nature of the streams. In the telecommunication sector, fractals are important to understand the radio signal and wavelength [4]. In addition, cryptography, encryption and image compression are also applications of the fractals[5]. Julia set is the collection of points where complex valued functions have chaotic behaviour and the collection of Julia sets of these functions is the Mandelbrot set. Every point of a fractal shows the same similarity as the entire, for which any appropriate focused part is more subdued when increased or decreased. The term “fractal” originates from the Latin word that signifies broken or fractured. Firstly Benoit Mandelbrot used the term fractal in 1978[1], and became the father of fractal geometry. For the generation of fractals, the role of the iterative scheme of fixed point theory is very important. Recently lots of authors used different iterative processes to generate the fractals for functions like sine,cosine,complex,exponential and so on[7]-[19].

In the present work, we use Dogan and Karakaya (DK) iterative process [20], to establish the escape criteria for generating fractals as Julia set and Mandelbrot set for logarithmic function $F(z) = log(1 + z^p) + c$, where $c \in \mathbb{C}$ and $p \geq 2$, we provide several examples to observe changes in generated graphical images and analyse the influence of underlying parameters on the dynamics, colour, generation time, and shape variations of produced fractals. The coloured points within the resulting fractals represent “escape points,” meaning they approach infinity through the DK-iterative method. The diverse colours indicate the rate at which a point escapes. Notably, some of our fractals bear a resemblance to traditional Kachhi Thread Works found in the Kutch district of Gujarat, India, proving beneficial in the Textile Industry and spinning wheel, cracker traditionally used during Diwali and Rangoli made in India contributing to interior decoration.

1. Preliminaries

We start with fundamental notions and definitions which are essential for this research.

Definition 2.1. ***Julia set*** [3, 21]***:*** *The collection of points within a set of complex numbers so that the trajectory of the function $F : \mathbb{C} \to \mathbb{C}$, diverging towards a point at infinity, is termed as filled Julia set of F. We write*

$$S_F = \{z \in \mathbb{C} : \{|F^i(z)|\}_{i=0}^{\infty} \text{ is bounded}\} \tag{1}$$

The Julia set of F is the boundary of S_F.

Definition 2.2. ***Mandelbrot set*** [22, 23]***:*** *The collection of parameters within a set of complex numbers so that the filled Julia set S_F of $F = z^2+c$ is connected is termed as Mandelbrot set. Mathematically it is defined as :*

$$M = \{c \in \mathbb{C}: S_F \text{ is connected}\}.$$

The Mandelbrot set M encapsulates significant information related to the Julia set and can also be expressed as:

$$M = \{c \in \mathbb{C}: |F(z)| \nrightarrow \infty \text{ as } k \to \infty\}. \quad (2)$$

Definition 2.3. ***Dogan and Karakaya (DK) iteration*** [20]***:****For $z_0 \in \mathbb{C}$, DK-iteration is defined as*

$$\begin{cases} z_{i+1} = (1-\alpha_i)F(x_i) + \alpha_i F(y_i), \\ y_i = (1-\beta_i)F(z_i) + \beta_i F(x_i), \\ x_i = F(z_i), \end{cases} \quad (3)$$

where $\{\alpha_i\}_{i=0}^{\infty}, \{\beta_i\}_{i=0}^{\infty} \in [0,1]$ and $i = 0,1,2....$

2. Main Results

To generate complex fractals escape criteria play an important role. In this section,we present the escape criterion for the logarithmic function via Dogan and Karakaya (DK) iteration[20]. Dogan and Karakaya(DK) iteration have three steps. In the first step, we have z_{i+1} which depends on y_i, In the second step, we have y_i which depends on x_i and in the final step, we have x_i which depends on z_i. The dependency on z_i extends to all x, y and z within the complex number set $\mathbb{C}$. Specifically, for $i = 0$, in this article, we make the assumption that $x_0 = x$, $y_0 = y$, and $z_0 = z$, and sequences $\{\alpha_i\}_{i=0}^{\infty}$ and $\{\beta_i\}_{i=0}^{\infty}$, are considered as constant sequences and we denote by $\{\alpha\}$ and $\{\beta\}$.

3.1. **Escape criteria of Dogan and Karakaya iteration for logarithmic function.**

Suppose $F(z) = log(1 + z^p) + c$ where $c \in$ C and $p \geq 2$ be a logarithmic function, then

$$\begin{aligned} |log(1+z^p)| &= |z^p - z^{2p}/2 + z^{3p}/3 - z^{4p}/4 \ldots| \\ &= |z^p||1 - z^p/2 + z^{2p}/3 - z^{3p}/4 \ldots| \quad (4) \\ &\geqslant |z^p||m_1|, \end{aligned}$$

Where $|m_1| \in (0, 1]$ satisfying the bound $|m_1| \leq |1 - z^p/2 + z^{2p}/3 - z^{3p}/4 \ldots|$; $z \in \mathbb{C}$, and similarly,

$$\begin{aligned} |log(1+y^p)| &= |y^p - y^{2p}/2 + y^{3p}/3 - y^{4p}/4 \ldots| \\ &= |y^p||1 - y^p/2 + y^{2p}/3 - y^{3p}/4 \ldots| \quad (5) \\ &\geqslant |y^p||m_2|, \end{aligned}$$

Where $|m_2| \in (0, 1]$ satisfying the bound $|m_2| \leq |1 - y^p/2 + y^{2p}/3 - y^{3p}/4 \ldots|$; $y \in \mathbb{C}$, and consequently,

$$\begin{aligned} |log(1+x^p)| &= |x^p - x^{2p}/2 + x^{3p}/3 - x^{4p}/4 \ldots| \\ &= |x^p||1 - x^p/2 + x^{2p}/3 - x^{3p}/4 \ldots| \quad (6) \\ &\geq |x^p||m_3|, \end{aligned}$$

Where $|m_3| \in (0, 1]$ satisfying the bound $|m_3| \leq |1 - x^p/2 + x^{2p}/3 - x^{3p}/4 \ldots|$; $x \in \mathbb{C}$.

Theorem 3.1. *Let* $F(z) = log(1 + z^p)$ *where* $p \geq 2$, $c \in \mathbb{C}$ *and* $\{z_i\}_{i\in W}$ *be the DK-iteration defined in* (3) *with* $|z| \geq |c| > \left[\frac{2}{|m_1|}\right]^{\frac{1}{(p-1)}}$, $|z| \geq |c| > \left[\frac{2}{\beta|m_3|}\right]^{\frac{1}{(p-1)}}$ *and* $|z| \geq |c| > \left[\frac{2}{(\alpha)|m_2|-|m_3|}\right]^{\frac{1}{(p-1)}}$. *Then* $|z| \to \infty$, *as* $i \to \infty$.

Proof. By fixing $x_0 = x$, $y_0 = y$ and $z_0 = z$, and from the given information, the first step of Dogan and Karakaya iteration

$$|x_i| = |F(z)| \Rightarrow |x_0| = |F(z_0)|.$$

Next, by using (4), and the given fact $|z| \geq |c| > \left[\frac{2}{|m_1|}\right]^{\frac{1}{(p-1)}}$

$\Rightarrow (|m_1||z^{p-1}| - 1) > 1,$

$$\begin{aligned}|x| = |F(z)| &= |log(1 + z^p) + c|\\ &\geq |log(1 + z^p)| - |c|\\ &\geq |m_1||z^p| - |z|;\ |z| \geq |c|\\ &\geq |z|\ |m_1||z^{p-1}| - 1\\ &\geq |z|.\end{aligned}$$

Next, in the second step of Dogan and Karakaya (DK) iteration,

$$|y_i| = |(1-\beta)F(z_i) + \beta F(x_i)| \Rightarrow |y_0| = |(1-\beta)F(z_0) + \beta F(x_0)|.$$

We have from (6), and $|x| \geq |z| \geq |c| > \left[\frac{2}{\beta|m_3|}\right]^{\frac{1}{p-1}} \Rightarrow (\beta|m_3||x^{p-1}|-1) > 1$, $|y| \geq |x|$, and

$$\begin{aligned}|y| &= |(1-\beta)F(z) + \beta F(x)|\\ &\geq |\beta F(x)| - |(1-\beta)x| \because x = F(z)\\ &\geq |\beta(log(1 + x^p) + c)| - (1-\beta)|x|\\ &\geq \beta|log(1 + x^p)| - \beta|c| - |x| + \beta|x|\\ &\geq \beta|m_3||x^p - \beta|z| - |x| + \beta|z|, \because |x| \geq |z| \geq |c|\\ &\geq \beta|m_3||x^p| - |x|\\ &\geq |x|(\beta|m_3||x^{p-1}| - 1)\\ &\geq |x|\end{aligned}$$

Next, the third step of the Dogan and Karakaya (DK) iteration scheme, we have

$$|z_{i+1}| = |(1-\alpha)F(x_i) + \alpha F(y_i)|$$

Taking i =0, we have

$$\begin{aligned}|z_1| &= |(1-\alpha)F(x_0) + \alpha F(y_0)|\\ &= |(1-\alpha)F(x) + \alpha F(y)|\\ &= |(1-\alpha)(log(1 + x^p) + c) + \alpha(log(1 + y^p) + c)|\\ &= |(1-\alpha)(log(1 + x^p)) + \alpha(log(1 + y^p)) + (1-\alpha)c + \alpha(c)|\\ &= |(1-\alpha)(log(1 + x^p)) + \alpha(log(1 + y^p)) + c|\\ &\geq \alpha|log(1 + y^p)| - |log(1 + x^p)| + \alpha|log(1 + x^p)| - |c|\\ &\geq \alpha|log(1 + y^p)| - |log(1 + x^p)| - |c| \quad (\text{Neglecting } \alpha|log(1 + x^p|)\\ &\geq \alpha|m_2||y^p| - |m_3||x^p| - |c|\\ &\geq \alpha|m_2||z^p| - |m_3||z^p| - |z| \quad (\because |y| \geq |x| \geq |z| \geq |c|)\\ &\geq (\alpha|m_2| - |m_3|)|z^p| - |z|\end{aligned}$$

$$|z_1| \geq |z|(\alpha|m_2| - |m_3|)|z^{p-1}| - 1)$$

Following the same pattern, for i=1,

$$|z_2| \geq |z|\ ((\alpha|m_2| - |m_3|)\ |z^{p-1}| - 1)^2.$$

Next, for i=2, we have

$$|z_3| \geq |z|\ ((\alpha|m_2| - |m_3|)\ |z^{p-1}| - 1)^3.$$

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$$|z_{i+1}| \geq |z|\ ((\alpha|m_2| - |m_3|)\ |z^{p-1}| - 1)^i.$$

Since $|z| \geq |c| > \left[\frac{2}{(\alpha)|m_2|-|m_3|}\right]^{\frac{1}{(p-1)}}$

$\Rightarrow$ *$(\alpha|m_2|-|m_3||z^{p-1}|-1) > 1$ and therefore,* $|z_i| \to \infty$ as i $\to \infty$.

Corollary 3.1. *For $k \geq 0$, if*

$$\left\{|z_i| > z_0 > \max\left\{|c|, \left[\left[\frac{2}{|m_1|}\right]^{\frac{1}{p-1}}\right], \left[\frac{2}{\beta|m_3|}\right]^{\frac{1}{p-1}}, \left[\frac{2}{(\alpha|m_2|-|m_3|)}\right]^{\frac{1}{p-1}}\right\}\right\}$$

then there exists a positive number $\theta > 0$ so that

$$(|z|\ (|m_1|)\ (\beta|m_3|)\ (\alpha|m_2|-|m_3|)\,|z^{p-1}|-1) > 1+\theta \Rightarrow |z_{k+i}| > |z_k|\,(1+\theta)^{k+i}$$

and then $|z_i| \to \infty$ as $i \to \infty$.

3. Visualization of Fractals as Julia and Mandelbrot sets

To visualize the Julia sets we use Algorithm 1 and for Mandelbrot sets, Algorithm 2, for obtaining fractals for logarithmic function using Dogan and Karakaya (DK) iteration scheme via MATLAB (R2015a) and colourmap (Figure 1). In this iterative process, numerous fractals emerge as Julia and Mandelbrot sets. It is noteworthy that many of these fractals exhibit symmetry. Some of our fractals exhibit similarities to the traditional Kachhi Thread Works found in the Kutch district of Gujarat, as well as Rangoli made in different parts of India. This resemblance proves advantageous in both the Textile Industry and interior decoration

1.1. **Julia set.** We visualize some Julia sets of function $F(z) = log(1+z^p) + c$ for different values of parameters using Dogan and Karakaya iteration. The maximum number of iterations we have considered is 30 (i.e., P=30).

Figure 1: Colourmap used in the visualization of fractals

Algorithm 1: For visualization of Julia Set

Input: $F(z) = log(1 + z^p) + c$, $c \in \mathbb{C}$; $p \geq 2$, $A \subset \mathbb{C}$ -area; P-maximum number of iterations; $\alpha, \beta \in (0, 1]$-parameter of Dogan and Karakaya iteration. Colourmap [0...C-1]- colour with C colours.

Output: Julia set for area A.

for $z_0 \in A$ **do**

R=Stopping threshold for Dogan and Karakaya iteration

i=0

while $i \leq P$ **do**

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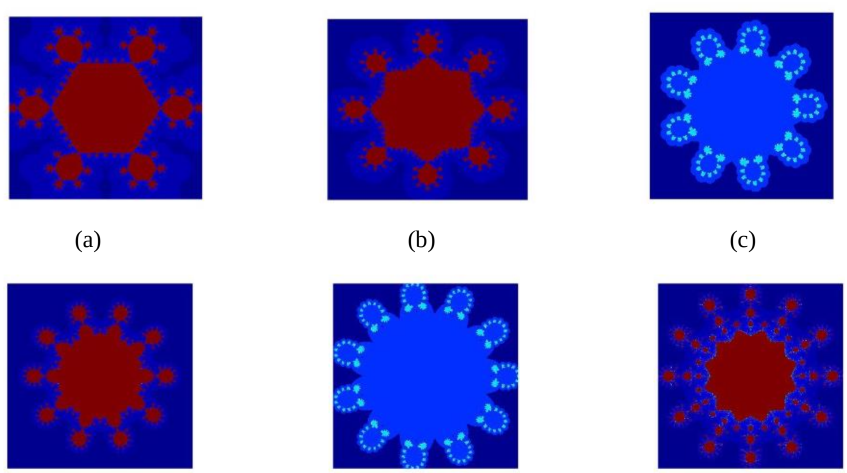
$z_{i+1} = (1-\alpha)F(x_i) + \alpha F(y_i)$,
$y_i = (1-\beta)F(z_i) + \beta F(x_i)$,
$x_i = F(z_i)$,
if $|z_{i+1}| > R$ **then**
break
end if
i=i+1
$j = [(C-1)\frac{i}{p}]$
colour z_0 with colourmap [i]
end for

Table 1: Parameter values for Figure 2.

Sr.No.	**c**	α	β	m_1	m_2	m_3	**p**
(a)	-1.1782	0.0025	0.0015	0.0075	0.0055	0.0025	6
(b)	-1.1782	0.0025	0.0015	0.0075	0.0055	0.0025	8
(c)	-1.1782	0.0025	0.0015	0.0075	0.0055	0.0025	9
(d)	-1.1782	0.0025	0.0015	0.0075	0.0055	0.0025	10
(e)	-1.1782	0.0025	0.0015	0.0075	0.0055	0.0025	11
(f)	-1.1782	0.0025	0.0015	0.0075	0.0055	0.0025	12



(a) (b) (c)

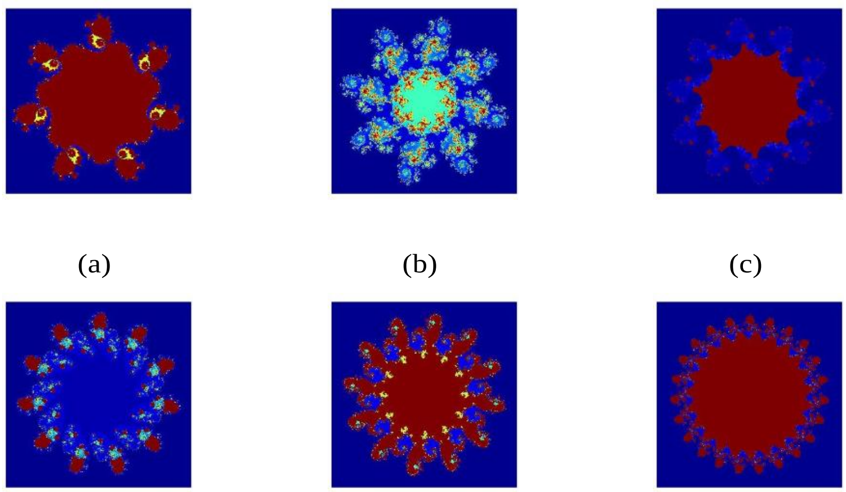
(d) (e) (f)

Figure 2. For the different values of p . **(a)** A = [-1.5,1.5]×[-1.5,1.5] and time 3.976422 Sec. **(b)** A = [-1.5,1.5]×[-1.5,1.5] and time 4.131868 Sec. **(c)** A = [-1.5,1.5]×[-1.5,1.5] and time 4.350221Sec. **(d)** A = [-1.5, 1.5]×[-1.5,1.5] and time 4.467810 Sec. **(e)** A =[-1.2,1.2]×[-1.2,1.2] and time 4.480143 Sec. **(f)** A = [-1.2,1.2]×[-1.2,1.2] and time 4.482672 Sec.(where A = Area).

We observe that Julia fractals appear like the traditional Kachhi Thread Works foundin the Kutch district of Gujarat, India, and the number of outer bulbs is equal to the value of p. Surprisingly, the color for odd values of p is blue, and for even values, it is red (See Table 1 and Figure 2).

Table 2: Parameter values for Figure 3.

Sr.No.	c	α	β	m_1	m_2	m_3	p
(a)	0.09-0.9i	0.2525	0.7779	0.3775	0.5995	0.0202	7
(b)	0.09-0.9i	0.2525	0.7779	0.3775	0.5995	0.0202	8
(c)	0.09-0.9i	0.2525	0.7779	0.3775	0.5995	0.0202	10
(d)	0.09-0.9i	0.2525	0.7779	0.3775	0.5995	0.0202	11
(e)	0.09-0.9i	0.2525	0.7779	0.3775	0.5995	0.0202	12
(f)	0.09-0.9i	0.2525	0.7779	0.3775	0.5995	0.0202	23



(a) (b) (c)

(d) (e) (f)

Figure 3. For different values of c and p . **(a)** A = [-1.2,1.2]×[-1.2,1.2] and time 4.544207 Sec. **(b)** A = [-1.2,1.2]×[-1.2,1.2] and time 4.471846 Sec. **(c)** A = [-1.2,1.2]×[-1.2,1.2] and time 4.752921 Sec. **(d)** A = [-1.2, 1.2]×[-1.2,1.2] and time 4.858966 Sec. **(e)** A = [-1.2, 1.2]×[-1.2,1.2] and time 4.738744 Sec. **(f)** A = [-1.2,1.2]×[-1.2,1.2] and time 5.479708 Sec.

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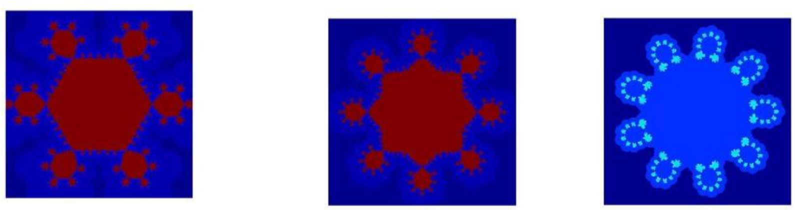
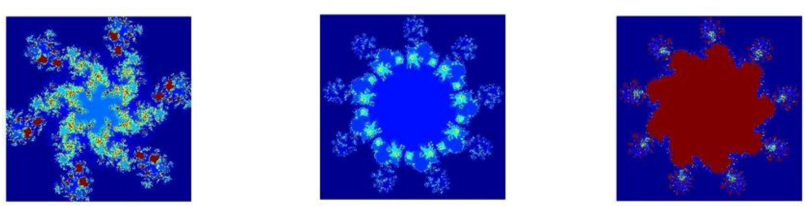
Julia fractals approach circular shapes as the value of p increases (See, Table 2 and Figure 3). These fractals share striking similarities with Rangoli patterns crafted in various regions of India. This likeness brings valuable advantages to the realm of interior decoration.

Table 3: Parameter values for Figure 4.

Sr.No.	**c**	α	β	m_1	m_2	m_3	**p**
(a)	0.01-i	0.3232	0.8859	0.5353	0.7070	0.4444	7
(b)	0.01-i	0.3232	0.8859	0.5353	0.7070	0.4444	11
(c)	-0.004+i	0.3232	0.8859	0.5353	0.7070	0.4444	7
(d)	-0.44+i	0.3232	0.8859	0.5353	0.7070	0.4444	6
(e)	-0.44+i	0.3232	0.8859	0.5353	0.7070	0.4444	9
(f)	-0.44+i	0.2525	0.7779	0.3775	0.5995	0.0202	9



(a) (b) (c)



(d) (e) (f)

Figure 4. For different values of c and p . **(a)** A = [-1.2,1.2]×[-1.2,1.2] and time 4.560058 Sec. **(b)** A = [-1.2,1.2]×[-1.2,1.2] and time 4.849844 Sec. **(c)** A = [-1.2,1.2]×[-1.2,1.2] and time 4.459713 Sec. **(d)** A = [-1.2,1.2]×[-1.2,1.2] and time 4.401038 Sec. **(e)** A = [- 1.2,1.2]×[-1.2,1.2] and time 4.759435 Sec.

Variation in distinct values of parameters for entries in Table 3 may be observed in Figure 4. Certain fractals within our collection bear a striking resemblance to the spinning wheel traditionally used during Diwali, as depicted in Figure 4(d).

Mandelbrot set. Here we discuss several Mandelbrot sets for the function $F(z) = log(1 + z^p) + c$ for different values of p in the trajectory of Dogan and Karakaya iteration. We have generated Mandelbrot sets for various parameters via Dogan and Karakaya (DK) iteration. For consistency across all fractals, we have set the maximum iteration to 30 (i.e., P=30) in Algorithm number 2.

Algorithm 2: For visualization of Mandelbrot Set

Input: $F(z) = log(1 + z^p) + c$, $c \in \mathbb{C}$; $p \geq 2, A \subset \mathbb{C}$ -area; P-maximum number of iterations; $\alpha, \beta \in (0, 1]$-parameter of Dogan and Karakaya iteration. Colourmap [0...C-1]- colour with C colours.

Output: Mandelbrot set for area A.

for $z_0 \in A$ **do**

R=Stopping threshold for Dogan and Karakaya iteration

i=0

while $i \leq P$ **do**

$z_{i+1} = (1-\alpha)F(x_i) + \alpha F(y_i)$,

$y_i = (1-\beta)F(z_i) + \beta F$

$(x_i), x_i = F(z_i)$,

if $|z_{i+1}| > R$ **then**

break

end if

i=i+1

$j = [(C-1)\frac{i}{p}]$

colour z_0 with colourmap [i]

end for

Table 4: Parameter values for Figure 5.

Sr.No.	α	β	m_1	m_2	m_3	**p**
(a)	0.05	0.02	0.05	0.09	0.05	5
(b)	0.05	0.02	0.05	0.09	0.05	6
(c)	0.05	0.02	0.05	0.09	0.05	7
(d)	0.05	0.02	0.05	0.09	0.05	8
(e)	0.05	0.02	0.05	0.09	0.05	9
(f)	0.05	0.02	0.05	0.09	0.05	10

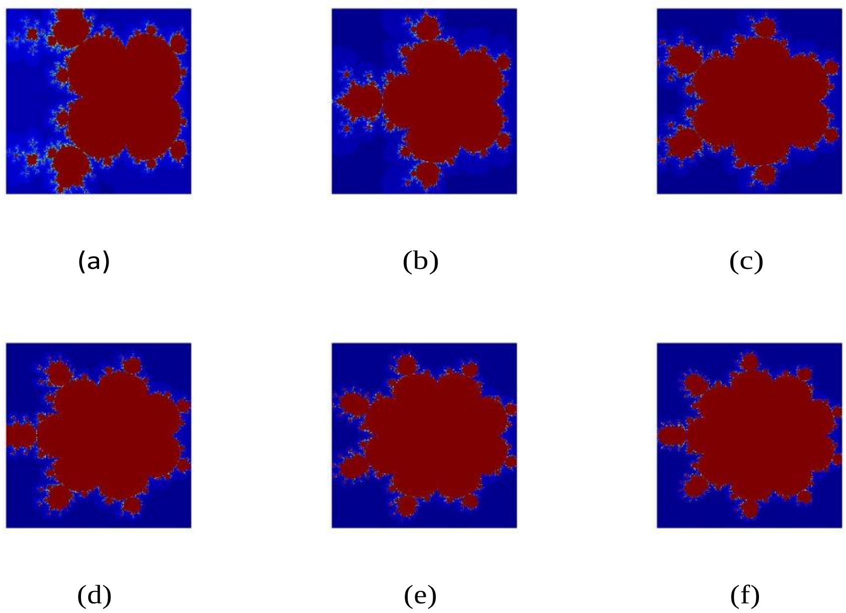
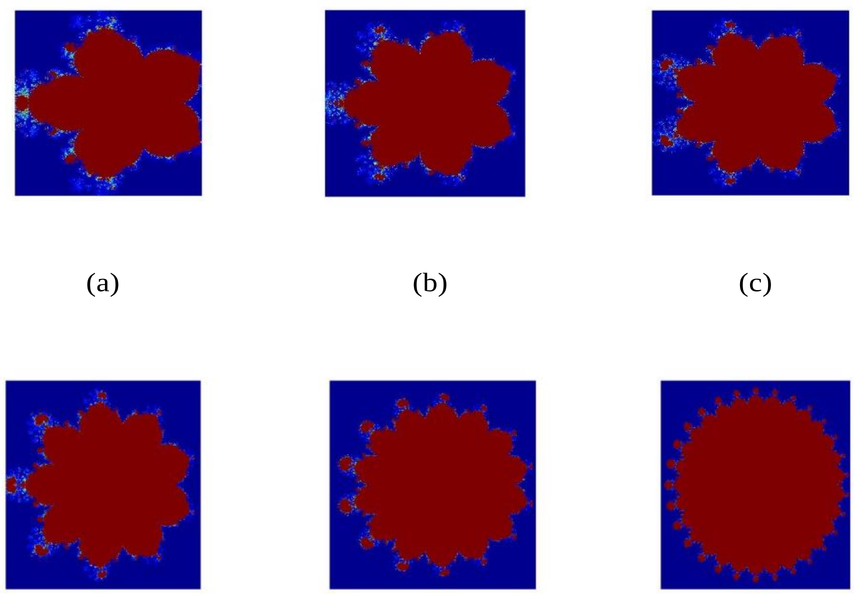


Figure 5. For different values of p effects on the Mandelbrot set in Dogan andKarakaya iteration. **(a)** A = [-1.5,0.8]×[-1,1.1] and time 4.076422 Sec. **(b)** A = [-1.5,0.9] ×[-1.2,1.2] and time 4.131868 Sec. **(c)** A = [-1.2,0.9]×[-1.2,1.2] and time 4.350221 Sec. **(d)** A = [-1.2,0.9]×[-1.2,1.2] and time 4.467810 s. **(e)** A = [-1.2,0.9]×[-1.2,1.2] and time 4.480143Sec. **(f)** A = [-1.2,0.9]×[-1.2,1.2] and time 4.482672 Sec.

Mandelbrot fractals appear as in Figure 5, and the number of outer bulbs is equal toone less than the numerical values of p for entries in Table 4.

Table 5: Parameter values for Figure 6.

Sr.No.	α	β	m_1	m_2	m_3	**p**
(a)	0.7777	0.6767	0.5555	0.0777	0.0003	6
(b)	0.7777	0.6767	0.5555	0.8659	0.3333	8
(c)	0.7777	0.6767	0.5555	0.8659	0.3333	9
(d)	0.7777	0.6767	0.5555	0.8659	0.3333	10
(e)	0.7777	0.6767	0.5555	0.8659	0.3333	15
(f)	0.7777	0.6767	0.5555	0.8659	0.3333	30



(a) (b) (c)

(d) (e) (f)

Figure 6. For different values of α, β and p effect on Mandelbrot set in Dogan and Karakaya iteration. **(a)** A = [-1.2 ,0.8]×[-1.2,1.2] and time 4.847634 Sec. **(b)** A = [-1.2,1]×[-1.2,1.2] and time 5.058047 Sec. **(c)** A = [-1.2,1]×[-1.2,1.2] and time 5.069966 Sec.**(d)** A=[-1.2,1]×[-1.2,1.2] and time 5.206482 Sec. **(e)** A= [-1.2,1]×[-1.2,1.2] and time 5.501968 Sec. **(f)**A= [-1.1,1]×[-1.1,1.1] and time 6.030451 Sec.

As the value of p increases, Mandelbrot fractals approach circular shapes. Refer to Figure 6 and Table 5 for details.

CONCLUSION

We have used Dogan and Karakaya(DK) iteration to prove escape criteria for the logarithmic function $F(z) = log(1 + z^p) + c$. Visualization of the Julia and Mandelbrot sets have been done by implementing the results in Algorithms 1 and 2 in Dogan Karakaya orbit. We have discussed the generated Julia sets and Mandelbrot sets for different parameters with detailed explanations. We also observed that for different values of p the number of attractors and repellers are different. Also for the complex value of c the fractals become more vibrant, and for the larger value of p, the Mandelbrot set starts having a circular shape. Further,we have calculated the execution time for generating the fractals in seconds which shows that for different parameters the execution time is different in generations of fractals as Julia and Mandelbrot set. We have explored variations in image generation and evaluated the impact of parameters on the dynamics, color, and overall appearance of fractals. Certain fractals in our collection exhibit a remarkable likeness to the traditional Kachhi Thread Works from the Kutch district of Gujarat, as well as Rangoli patterns made in different parts of India. This similarity offers significant advantages in both the Textile Industry and the field of interior decoration.

Additionally, specific fractals in our collection showcase a striking resemblance to the spinning wheel traditionally used during Diwali, as illustrated. These findings can be used to find the application of different iterative schemes in the generation of fractals.

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